NONSTEADY HEAT EXCHANGE DURING PULSATING LAMINAR FLOW OF A VISCOUS LIQUID IN AN ANNULAR CHANNEL

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Problems of the active action (mechanical, acoustic, or electromagnetic, for a conducting medium) on a flow, having the purpose of the intensification of heat exchange [1], have recently elicited the interest of investigators. The mechanical methods of action include the use of pulsating streams of coolants to increase the heat removal in the thermal initial sections of channels with laminar nonsteady flow of a viscous incompressible liquid.

The possibility of intensifying heat exchange by varying the frequency and amplitude of pulsations in the pressure gradient in annular channels is investigated in the present report, since there are presently conflicting data on this problem in a number of reports [2-5]. The choice of an annular channel is explained by the wide distribution in engineering of heat-exchange devices formed by two coaxial cylinders, as well as by the absence of theoretical and experimental reports on heat transfer during pulsating flow in channels of such shape. In contrast to the known reports, the problem is solved in a conjugate formulation, which most fully reflects the real nonsteady thermal processes in the flow of a viscous liquid in the thermal initial section when the conditions at the interfaces of the media are not known in advance and the temperatures in the walls and the liquid must be determined jointly [6, 7]. The influence of the dissipative function and the axial heat conduction both in the liquid and in the channel walls is taken into account. This allows one to apply the methods developed for the solution to the calculation of conjugate heat transfer through a coolant with arbitrary Prandtl numbers.

The conjugate formulation of the problem makes it possible to estimate the heat-exchange intensity using the heat flux summed over several pulsation periods ($Q=\int_0^{t_*} q|_\Gamma dt$ over the time of establishment of the process or $Q=\int_{t_1}^{t_2} q|_\Gamma dt$ over equal time intervals for different frequencies in the established mode; q is the specific heat flux) rather than by comparing average Nusselt numbers for different modes of flow [2, 3], since when there is nonsteady longitudinal nonisothermicity in the channel walls, e.g., the use of Nusselt numbers loses physical meaning [8].

1. Pulsating flows can be arbitrarily divided into two types: 1) the imposition of pulsations on the main flow [$\partial p/\partial x = \partial p/\partial x\,|_0 + f(t)$, with f(t) being a periodic function of time]; 2) "pure" pulsations ($\partial p/\partial x\,|_0 \equiv 0$).

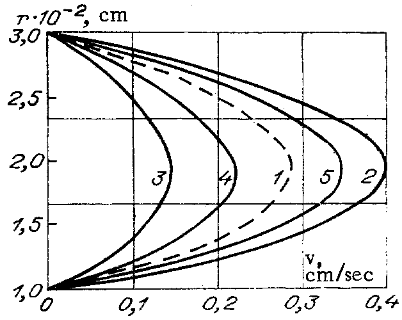

Fig. 1

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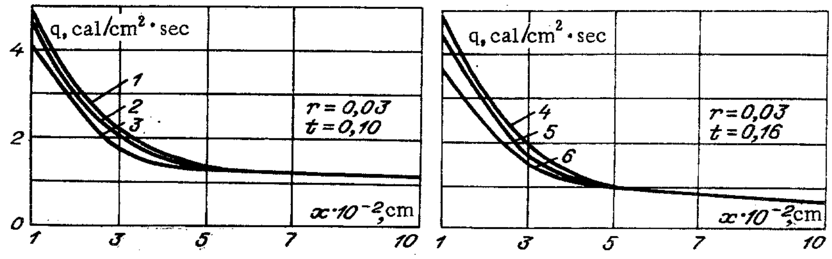

Fig. 2

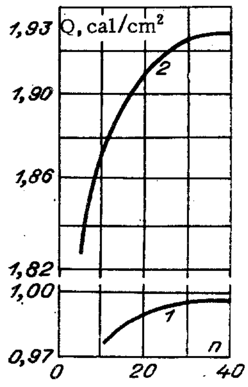

Fig. 3

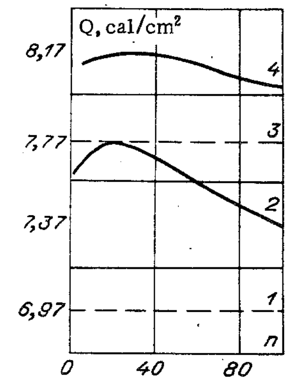

Fig. 4

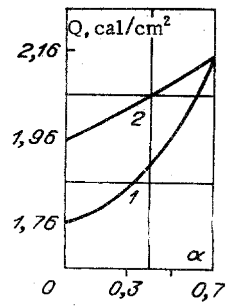

Fig. 5

The comparison of heat-exchange intensity in the first case must be made with allowance for the equal liquid flow rate for $\partial p/\partial x = \mathrm{const}$ and $\partial p/\partial x = F(t)$; in the second case it is desirable to compare boundary heat fluxes for different pulsation frequencies. We investigated both types of flows with frequencies $n = \omega/2\pi = 2\text{–}100\ \mathrm{sec}^{-1}$, since the calculations showed that higher frequencies do not have an appreciable effect on the heat transfer (under the conditions analyzed in the report).

The process of flow and heat exchange is described by a set of equations and boundary conditions in the form

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right), \quad \nu = \mu/\rho; \tag{1.1}$$

$$\frac{\partial T_i}{\partial t} = a_i\left(\frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r}\frac{\partial T_i}{\partial r}\right) + W_i, \quad i = 1, 2, 3; \tag{1.2}$$

$$W_1 = W_3 = 0, \quad W_2 = \frac{\nu}{c_p}\left(\frac{\partial v}{\partial r}\right)^2 - v\frac{\partial T_2}{\partial x}; \tag{1.3}$$

$$\text{at} \quad x = 0 \quad T_i = T_+; \quad \left.\frac{\partial T_i}{\partial x}\right|_{x\to\infty} = 0, \quad i = 1, 2, 3; \tag{1.4}$$

$$\text{at} \quad r = 0 \quad \frac{\partial T_1}{\partial r} = 0; \quad \text{at} \quad r = r_3 \quad T_3 = T_\infty; \tag{1.5}$$

$$\text{at} \quad r = r_1 \quad v = 0, \quad T_1 = T_2, \quad \lambda_1\frac{\partial T_1}{\partial r} = \lambda_2\frac{\partial T_2}{\partial r}; \tag{1.6}$$

$$\text{at} \quad r = r_2 \quad v = 0, \quad T_2 = T_3, \quad \lambda_2\frac{\partial T_2}{\partial r} = \lambda_3\frac{\partial T_3}{\partial r}; \tag{1.7}$$

$$\text{at} \quad t = 0 \quad T_1 = T_2 = T_0, \quad T_3 = T_\infty, \quad v = v_0; \tag{1.8}$$

$$-\frac{\partial p}{\partial x} = B + A\sin\omega t, \quad A < B, \tag{1.9}$$

$$v_0 = Br_1^2\frac{1}{4\mu}\left[\frac{(r_2/r_1)^2 - 1}{\ln(r_2/r_1)}\ln\frac{r}{r_1} - \frac{r^2}{r_1^2} + 1\right];$$

$$-\frac{\partial p}{\partial x} = A\sin\omega t, \quad v_0 = 0, \tag{1.10}$$

where v is the velocity of the liquid; T_i, $i = 1, 2, 3$ are the temperatures of the inner cylinder, the liquid, and the wall, respectively; x, r, and t are the cylindrical coordinates and time; λ_i, a_i, $i = 1, 2, 3$ are the coefficients of thermal conductivity and diffusivity; ρ, c_p, and μ are the density, specific heat capacity, and coefficient of viscosity of the liquid; T_+ is the temperature at the inlet; T_∞ is the temperature of the medium; ω and A are the angular frequency and the amplitude of the pulsations.

Equations (1.9) correspond to the case of pulsations with imposition on the main flow and (1.10) correspond to "pure" pulsations.

2. The constancy of the thermophysical properties of the liquid and the channel walls makes it possible to solve the hydrodynamic and thermal parts of the problem separately.

The velocity distribution was obtained analytically by the method of finite integral transformations with respect to the r coordinate [9] in the form

$$v(r, t) = \sum_{n=1}^{\infty} \frac{V_n(r)}{H_n} v_{\mathrm{I}}(\beta_n, t),$$

where $v_{\mathrm{I}}(\beta_n, t) = \exp(-a\beta_n^2 t)\left[f_{\mathrm{I}}(\beta_n) + a\int_0^t e^{a\beta_n^2 t} Q_{\mathrm{I}}(\beta_n, t)\,dt\right]$; $V_n(r) = J_0(\beta_n r) - \frac{J_0(\beta_n r_1)}{N_0(\beta_n r_1)} N_0(\beta_n r)$; $H_n = \|V_n\|^2$;

β_n are the roots of the characteristic equation; $J_0(\beta r_2)N_0(\beta r_1) - J_0(\beta r_1)N_0(\beta r_2) = 0$; $f_{\mathrm{I}}(\beta_n)$ and $Q_{\mathrm{I}}(\beta_n, t)$ are known functions; J_0 and N_0 are zeroth-order Bessel functions of the first and second kinds, respectively.

The solution of the thermal problem is constructed from the value found for the velocity:

1. The numerical solution is based on the scheme of decomposing Eqs. (1.2) into six one-dimensional equations, the approximation of the latter by an implicit two-layer scheme, and the solution of the finite-difference equations using the trial-run method [10]. For a simple realization of the trial-run method it becomes necessary to assign certain limits on T_i at $x = l > d$, where d is the equivalent diameter of the channel, in place of the second condition of (1.4). In the process of the calculation we used the conditions $\partial T_i/\partial x = 0$ or the "soft" boundary conditions $\partial^2 T_i/\partial x^2 = 0$.

2. A semianalytical solution was obtained using a Laplace transformation with respect to the longitudinal x coordinate for Eqs. (1.2) modified with $\lambda_i \frac{\partial^2 T_i}{\partial x^2} \ll \lambda_i \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_i}{\partial r}\right)$ in the absence of the second of the conditions (1.4) and an approximation of the resulting equations by an implicit scheme with the subsequent use of a trial run to solve the finite-difference equations in the transform space. The transition back to the inverse-transform space is made by the method of [11] by the equation

$$\varphi(\theta) = \sum_{n=0}^{\infty} c_n(r, t)\sin(2n+1)\theta, \quad \theta = \arccos e^{-\sigma x},$$

and $s = (2n+1)\sigma$ is the transform parameter.

3. In Figs. 1-5 we present the results of calculations for the following concrete cases: the liquid is water, the inner cylinder is steel, $\alpha = A/B = 0.3\text{-}0.7$, and $\Pr = \nu/a_2 = 12.5$; variant I: copper wall, $T_\infty = 303$°K, $T_+ = T_0 = 273$°K, $\mathrm{Re} = \langle v_0\rangle d/\nu = 0.4$, $\mathrm{Sh} = nd/\langle v_0\rangle = 0.8\text{-}16$, $\mathrm{Eu} = Bd/\rho\langle v_0\rangle^2 = 400$, and $\mathrm{Br} = \mu\langle v_0\rangle^2/\lambda_2(T_\infty - T_+) = 0.0166$; variant II: steel wall, $T_\infty = 273$°K, $T_+ = T_0 = 303$°K, Re = 1.2, Sh = 0.034-1.7, Eu = 36.4, and Br = −0.612 (Re, Pr, Sh, Eu, and Br are the Reynolds, Prandtl, Strouhal, Euler, and Brinkmann numbers, respectively).

In Fig. 1 we present velocity profiles for the case of pulsations with imposition on the main flow at different times for different frequencies with B = 100 and A = 50: 1) t = 0; 2) t = 0.06, n = 20; 3) t = 0.09, n = 20; 4) t = 0.06, n = 10; 5) t = 0.11, n = 10 (variant I). It is seen that velocity fluctuations near the initial profile are characteristic when the load disturbances are assigned in the form of (1.9). Strong return flows develop in the case of pure pulsations (B = 0).

The heat flux $q = -\lambda\partial T/\partial r|_{r=r_2}$ at the wall as a function of the longitudinal x coordinate at different times for $\alpha = 0.5$ is shown in Fig. 2: 1) n = 5; 2) n = 0 (steady flow); 3) n = 20; 4) n = 2.5; 5) n = 20; 6) n = 5 (variant I).

The total heat flux $Q = \int_0^{t_*} q|_{r=r_2} dt$ as a function of the frequency n and amplitude A of the pulsations is presented in Figs. 3-5; Fig. 3: x = 0.01, α = 0.5; 1) t_* = 0.2; 2) t_* = 0.4 (variant I); Fig. 4: α = 0.5, t_* = 0.4; 1) x = 0.05, n = 0; 2) x = 0.05; 3) x = 0.01, n = 0; 4) x = 0.01 (variant II); Fig. 5: n = 10, t_* = 0.1; 1) x = 0.05; 2) x = 0.01 (variant II).

The instantaneous heat fluxes at different times can differ by tens of percent for different frequencies, considerably exceeding the values of q for steady flow. These results are in qualitative agreement with the experimental results of [5].

At small values of Br (a weak influence of energy dissipation) the total heat fluxes increase with an increase in frequency, both for the imposition of pulsations on the main flow and for pure pulsations, with the rate of increase in the total heat removal slowing with an increase in frequency. For long enough times the total heat removal asymptotically approaches the heat removal for steady flow with an increase in frequency. It should be noted, however, that the heat transfer is strengthened insignificantly (by 5%).

With an increase in Br (variant II) the total heat removal increases by more than 10% in comparison with nonpulsating flow and by more than 20% with variation in A (see Figs. 4 and 5). Under the conditions analyzed in the report the maximum increase in heat transfer is observed in the frequency interval of n = 20-30 sec^{-1}.

Thus, in the case of negligibly low energy dissipation one observes an insignificant intensification of heat exchange by pulsations in the pressure gradient. But pulsating flows of coolants should not be considered as a replacement for steady flows with the same liquid flow rate. They are an independent flexible means of control of heat exchange, in chemical technology apparatus (in dissolution, drying, etc.), e.g., where processes take place more intensely in the presence of pulsations.

When dissipative effects are important there is a significant increase in heat removal in comparison with nonpulsating flow. This allows one to use pulsating streams in the indicated case for the intensification of the process of heat exchange in channels.

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LITERATURE CITED

1. A. E. Bergles, "Recent developments in convective heat-transfer augmentation," Appl. Mech. Rev., 26, No. 6, 675-682 (1973).
2. M. Sadanari and H. Keizo, "Heat exchange in pulsating flow of a liquid in a horizontal pipe. Part 1. Theoretical analysis," Trans. Jpn. Soc. Mech. Eng., 39, No. 318, 682-691 (1973).
3. J. Hapke, "Wärmeübergang bei pulsierender laminarer Strömung," Brennst. Wärme-Kraft, 26, No. 2, 55-63 (1974).
4. M. F. Edwards, D. A. Nellist, and W. L. Wilkinson, "Heat-transfer to viscous fluids in pulsating flow in pipes," Chem. Eng., No. 279, 532-537 (1973).
5. Yu. G. Minakov, "Critical heat fluxes in pulsating water flow in pipes," Izv. Vyssh. Uchebn. Zaved. Aviats. Tekh., No. 2, 120-125 (1968).
6. A. V. Lykov and T. L. Perel'man, "Nonsteady heat exchange between a body and a fluid stream flowing over it," in: Heat and Mass Exchange of Bodies with a Surrounding Gaseous Medium [in Russian], Nauka i Tekhnika, Minsk (1965).
7. V. I. Kondrashov and V. E. Tomilov, "Nonsteady heat exchange in laminar fluid flow in channels," in: Numerical Methods in the Mechanics of a Continuous Medium [in Russian], Vol. 7, No. 6, Izd. Vychis. Tsentr. Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1976).
8. L. V. Kim and V. I. Kondrashov, "Nonsteady heat exchange in the thermal initial section of a flat channel," in: Materials of the Practical Science Conference "Young Scientists of the Tomsk Oblast' in the Ninth Five-Year Plan [in Russian], Izd. Tomsk, Univ., Tomsk (1975).
9. G. A. Grinberg, Selected Problems of the Mathematical Theory of Electrical and Magnetic Phenomena [in Russian], Izd. Akad. Nauk SSSR, Moscow–Leningrad (1948).
10. S. D. Godunov and V. S. Ryaben'kii, Difference Schemes [in Russian], Nauka, Moscow (1973).
11. A. Papoulis, "A new method of inversion of the Laplace transform," Q. Appl. Math., 14, No. 4, 405-414 (1957).